Take Home Assignment: CY1001: Statistical Thermodynamics; 30 September 2013

1. N independent particles exist in one of the 3 non-degenerate energy levels of energies −E, 0, +E. The system is in contact with a thermal reservoir at temperature T. What is the partition function of the system? Use canonical ensemble to show that
(i) The maximum possible entropy in the limit $T \rightarrow 1$ is S = Nk ln 3
(ii) The minimum possible energy in the limit $T \rightarrow 0$ is E = −NE

2. Consider a system of distinguishable particles having only two non-degenerate energy levels separated by an energy which is equal to the value of kT at 10K. Calculate at 10K (a) the ratio of populations in the two states (b) the molecular partition function (c) the molar energy (d) the molar heat capacity (e) the molar entropy.

3. Calculate the translational partition function of an H_2 molecule confined to a 100 cm^3 vessel at 25°C.

4. An electron spin can adopt either of two orientations in a magnetic field, and its energies are $\pm\mu_B \beta$, where μ_B is the Bohr magneton (a) Deduce an expression for the partition function and mean energy of the electron. (b) Calculate the relative populations of the spin states at 4K and $\beta = 1.0T$.

5. Calculate the fraction of $N_2(g)$ molecules in the v = 0 and v = 1 vibrational states at 300K, given that $h\nu/k_B = 3374K$.

6. The molecules of a gas have two states of internal energy with statistical weights g_1, g_2 and energies 0, ε respectively. Calculate the contribution of these states to the specific heat of the gas.

7. A gas is composed of three atoms of Xe with atomic partition function, q_{Xe} and four atoms of Ar with atomic partition function, q_{Ar}. Write down the total partition function for this system.

8. The entropy of a monoatomic ideal gas is given by the following equation

$$S = Nk_B \left[\ln \left\{ \frac{V}{N} \left(\frac{4\pi mU}{3Nh^2} \right)^{\frac{3}{2}} \right\} + \frac{5}{2} \right]$$ Using the fundamental equation

dU = TdS – PdV, obtain the ideal gas law from the above equation.