

Department of Chemistry
Indian Institute of Technology Madras
CY 101 Assignment 2

November 9, 2009

Principal issues discussed:

Carnot engine

Definition of entropy

Entropy calculations – reversible, irreversible

Clausius inequality, conditions of equilibrium A, G

Gibbs free energy

Variation of G, $VdP-SdT$

Maxwell's relations

Phase equilibrium

Boltzmann distribution

Molecular partition function, U

Canonical partition function

Thermodynamic functions

Translational, vibrational, rotational and electronic partition functions

1. Find the melting point of ice at 100 atm if $\Delta H_{\text{fusion}} = 79.7 \text{ cal gm}^{-1}$. $\rho = 0.917 \text{ gm cm}^{-3}$ for ice at 0°C and 1 atm and $\rho = 1.00 \text{ gm cm}^{-3}$ for water at 1 atm.

(Ans : 272.4 K)

Ans: $\Delta H_{\text{fusion}} = 79.7 \text{ cal gm}^{-1}$

$$\Delta V_{\text{fusion}} = V_{\text{liq}} - V_{\text{solid}} \\ = -0.091 \text{ cm}^3$$

$$\ln T_2/T_1 = (P_2 - P_1) (\Delta V_{\text{fusion}}) / \Delta H_{\text{fusion}}$$

After substitution, $T_2 \simeq 272.4 \text{ K}$.

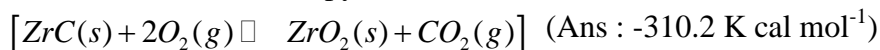
2. What is the change in the boiling point of H_2O at 100°C per mm change in the atmospheric pressure?. The heat of vapourisation is $539.7 \text{ cal gm}^{-1}$. The molar volume of $\text{H}_2\text{O} (\text{l})$ is 18.78 ml and that of $\text{H}_2\text{O} (\text{g})$ is 30.199 lit. (at 100°C and 1 atm) (Ans :

$$\frac{dT}{dP} = .037 \text{ K mm}^1)$$

$$\text{Ans: } dP/dT = \Delta H_{\text{vap}}/T (V_v - V_l) \\ = 27.1 \text{ mm Hg K}^{-1}.$$

$$\therefore dT/dP = 0.037 \text{ K mm Hg}^{-1} \text{ (K/Torr)}$$

3. The internal energy change (ΔU) for the combustion of $ZrC(s)$ was -310863 cal/mol at 25°C . Calculate the enthalpy of combustion for



$$\text{Ans: } \Delta H = \Delta U + (\Delta n) RT$$

$$\Delta U = \Delta H - (\Delta n) RT$$

$$= -310863 - (-1) \times 2 \times 298$$

$$\approx -310.2 \text{ k cal mol}^{-1}$$

4. The normal boiling point of C_6H_6 at 1 atm pressure is 353.2 K. Estimate the pressure at which C_6H_6 would boil at 330 K. (Ans : 0.432 atm)

Ans: To obtain ΔH_v , employ Trouton's rule

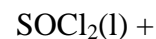
$$\therefore \Delta H_v \approx 31800 \text{ J/mol}$$

$$\ln P_2/P_1 = 31800 (330 - 353.2) / (8.314) (353.2) (330)$$

$$\therefore P_2 \approx 0.432 \text{ atm.}$$

Note: Depending upon the value of ΔH_v employed, the answer for P_2 may vary slightly.

5. Evaluate ΔG^0 , ΔH^0 , and ΔS^0 at 298 K for the reaction



$H_2O \rightarrow SO_2(g) + 2HCl(g)$ from the following data

Quantity	$SOCl_2(l)$	$H_2O(l)$	$SO_2(g)$	$HCl(g)$
$S_{298}^0 (\text{J mol}^{-1} \text{K}^{-1})$	215.7	69.9	248.1	186.8
$\Delta H_f^0 \text{ at } 298 \text{ K } (\text{kJ mol}^{-1})$	-245.6	-241.8	-296.8	-92.3

$$(\text{Ans} : \Delta S_{298}^0 = 336.1 \text{ J mol}^{-1} \text{K}^{-1};$$

$$\Delta H_{298}^0 = 6 \text{ kJ mol}^{-1}, \Delta G_{298}^0 = -94.15 \text{ kJ mol}^{-1}$$

Ans: ΔG^0 , ΔH^0 , ΔS^0 for the reaction

$$\Delta H_{298}^0 = [- (296.8) + 2 (-92.3)] - [-245.6 - 241.8]$$

$$= 6 \text{ kJ / mol.}$$

$$\Delta S_{298}^0 = 336.1 \text{ J K}^{-1} \text{ mol}^{-1}; \Delta G_{298}^0 = 6000 - (298) (336.1) = -94.15 \text{ kJ mol}^{-1}$$

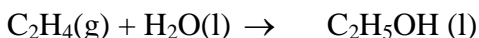
6. Calculate the change in the free energy of 1 mole of acetylene when it is heated from 500K to 600 K at constant pressure. The average entropy of acetylene in this temperature change is 230 J/K/mol. (Ans : 23 kJ/mol).

$$\text{Ans: } dG = VdP - SdT$$

$$\Delta G = -S(T_2 - T_1)$$

$$= -230 (600 - 500) = -2300 \text{ J/mol.}$$

7. From the given data predict whether the reaction



is spontaneous.

	ΔH_f^0 (kJ/mol)	S^0 (J/K/mol)
C_2H_5OH (l)	-277.69	160.7
H_2O (l)	-285.83	69.91
C_2H_4 (g)	52.26	291.56

(Ans: 15.7 kJ. nonspontaneous)

$$\text{Ans: } \Delta S^0 = 160.7 - (69.91 + 291.56)$$

$$= -200.77 \text{ JK}^{-1}\text{mol}^{-1}$$

$$\Delta H^0 = -277.69 - (-285.83 + 52.26)$$

$$= -44.12 \text{ kJ mol}^{-1}$$

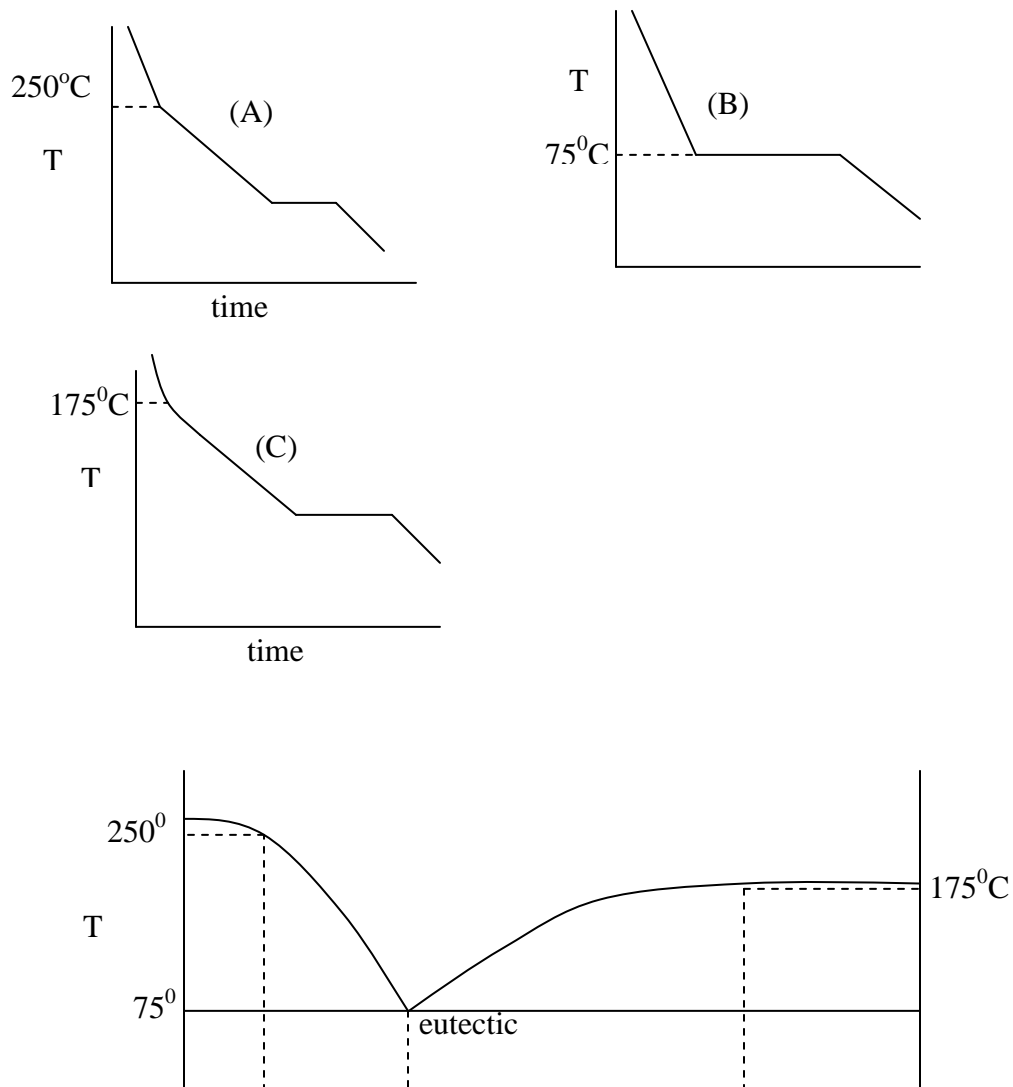
$$\Delta G^0 = -44120 - (298)(-200.77)$$

$$= 15.709 \text{ J mol}^{-1}$$

= + ve \therefore non-spontaneous in the direction specified.

10. Construct the phase diagram of a two component system forming simple eutectic. Apply the phase rule in each region. Draw the cooling curve for pure A, at eutectic composition and another one in between these two compositions.

Ans: Phase diagram of a two - component system forming a simple eutectic.



12. The vapour pressure in torr for ice and liq. water are given by the equations. (energy in joules)

$$\ln P (\text{ice}) = 24.00 - \frac{6140.1}{T}$$

$$\ln P (\text{H}_2\text{O}) = 21.41 - \frac{5433}{T}$$

Calculate the triple point temperature and pressure. Determine the molar enthalpies of vapourisation, sublimation and fusion of H₂O at its triple point

(Ans : 273.01 K & 4.52 torr; 45.2, 51.1 & 5.9 kJ mol⁻¹)

Ans: At the triple point temperature,

$$24.00 - 6140.1 / T = 21.41 - 5433 / T$$

$$\therefore 6140.1/T - 5433 / T = 2.59$$

$$707.1 / T = 2.59$$

$$\therefore T = 273.01 \text{ K.}$$

Substituting in any of the equations, we obtain the triple point pressure as

$$\ln P = 24.00 - 6140.1/273.01$$

$$= 1.509$$

$$\therefore P \simeq 4.52 \text{ Torr.}$$

From the first eqn, $d \ln p/dT = 6140.1/T^2 = \Delta H^0 (\text{sublimation})/RT^2$

$$\therefore \Delta H^0_{\text{sub}} = 6140.1 \times 8.314 = 51.05 \text{ kJ mol}^{-1}.$$

From the second eqn, $d \ln p/dT = 5433 / T^2 = \Delta H^0_{\text{vap}}/RT^2$

$$\therefore \Delta H^0_{\text{vap}} = 45.17 \text{ kJ mol}^{-1}$$

$$\therefore \Delta H^0_{\text{fusion}} = 51.05 - 45.17 = 5.88 \text{ kJ mol}^{-1}.$$

13. Calculate the change in entropy of a mole of aluminium (at.wt .27) which is heated from 600 to 700⁰ C. The m.pt .of Al is 660⁰ C, the heat of fusion is 393 Jg⁻¹ and the heat capacities of the solid and liquid may be taken as 31.8 amd 34.3 JK⁻¹ mol respectively. (Ans : 14.92 JK⁻¹ mol⁻¹)

$$\text{Ans: } \Delta S = C_p^s \ln T_m/T_1 + \Delta H_f/T_m + C_p^l \ln T_2/T_m$$

$$\Delta H_f = 393 \text{ J gm}^{-1} = 10.611 \text{ kJ mol}^{-1}.$$

$$\therefore \Delta S = 31.8 \ln 933/873 + 10611/933 + 34.4 \ln 973/933$$

$$\simeq 14.93 \text{ JK}^{-1} \text{ mol}^{-1}.$$

14. Based on the Trouton's rule, calculate the molar heat of vapourisation of benzene at its boiling point of 353 K (Ans: 31 kJ)

$$\text{Ans: } \Delta H^0 = 90 \text{ J mol}^{-1} \quad \therefore \Delta H_v^0 = 90 \times 353 \approx 31.77 \text{ kJ}$$

15. Calculate the molar heat capacity of a non – linear triatomic gas assuming that the principle of equipartition is valid (Ans : 49.9 J mol⁻¹ deg⁻¹)

$$\text{Ans: } C_v = 3R/2 + 3R/2 + 3R \approx 49.88 \text{ JK}^{-1} \text{ mol}^{-1}.$$

17. 100 g N₂ at 300 K were held by a piston at 30 atm. Pressure was released suddenly to become 10 atm, adiabatically. Calculate ΔS . $C_v = 20.8 \text{ JK}^{-1} \text{ mol}^{-1}$.

$$\Delta S = n(C_v \ln T_2/T_1 + R \ln V_2/V_1)$$

Sudden adiabatic expansion, T_2 is unknown.

$$V_2/V_1 = T_2 P_1 / T_1 P_2 = 217.8$$

$$\Delta S = -0.692 \text{ JK}^{-1}$$

18. Calculate entropy change when 0.5 L ideal gas, $C_v = 12.6 \text{ JK}^{-1} \text{ mol}^{-1}$, at 300 K and 1 atm was allowed to expand to double its volume while heated simultaneously to 373 K.

$$\Delta S = n(C_v \ln T_2/T_1 + R \ln V_2/V_1)$$

$$\Delta S = 0.17 \text{ JK}^{-1}$$

19. 10 g ice at 0 oC are added to 20 g water at 90 oC, in a thermally insulated flask of negligible heat capacity. Heat of fusion of ice is 6 kJ mol⁻¹. Calculate the final temperature and the entropy change of the system. $C_p = 75.42 \text{ JK}^{-1} \text{ mol}^{-1}$.

10 g H₂O (s), 273 K \rightarrow H₂O(l) 273 K \rightarrow H₂O(l), T K , Two entropy changes, ΔS_1 , ΔS_2
 90 g H₂O(l) 363 K \rightarrow H₂O(l), T K, ΔS_3

$$\text{Heat balance equation, } nL_f + C_p(T-273) = nC_p(363-T); T = 306.5$$

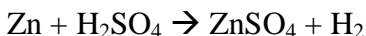
$$\Delta S_1 = nL_f/T = 12.2 \text{ JK}^{-1}$$

$$\Delta S_2 = n(C_p \ln T_2/T_1) = 4.85 \text{ JK}^{-1}$$

$$\Delta S_3 = n(C_p \ln T_2/T_1) = -14.31 \text{ JK}^{-1}$$

$$\Delta S = \text{sum} = 2.75 \text{ JK}^{-1}, \text{ corresponds only to the system}$$

20. In an open beaker held at 27 oC and 1 at pressure containing dilute sulphuric acid, 100 g zinc were added. Calculate the work done by the liberated gas. Calculate the work done if the reaction was carried out in a closed container.



$$W_{\text{irr}} = \Delta nRT = 100/65 \times 8.314 \times 300 = 3.84 \text{ kJ}$$

Closed container, no volume change, no work.

21. A balloon containing air at 1 atm pressure is 0.5 m in diameter. It was isothermally filled with air so that the pressure is 5 atm. Assuming the pressure is proportional to the diameter of the balloon, calculate the (a) final diameter and (b) work done in the process.

$P \propto d$

$$d_{\text{final}} = 2.5 \text{ m}$$

$$P = kd = k = 2 \text{ atm m}^{-1}$$

$$dW = P dV = kd \cdot 3(4\pi)(d/2)^2 dd = k\pi/2 d^3 dd$$

$$W = k\pi/2 \int_{0.5}^{2.5} d^3 dd = k\pi/8 [2.5^4 - 0.5^4] = [2 \text{ atm m}^{-1} \times \pi/8] \times 39 = 30.63 \text{ atm m}^3 \\ = 3.1 \times 10^6 \text{ J}$$

22. The entropy change of argon is given to a good approximation by the expression, $S \text{ JK}^{-1} \text{ mol}^{-1} = 36.36 + 20.79 \ln T$

Calculate change in Gibb's free energy of one mole of argon gas if it is heated at constant pressure from 25 °C to 50 °C.

$$dG = VdP - SdT, \text{ at constant } P, dG = -SdT$$

$$\int dG = -[36.36 \int dT + 20.79 \int \ln T dT] = -36.36 (T_2 - T_1) - 20.79 [T \ln T - T]_{T_1}^{T_2} \\ = -999 - 20.79 (1543.18 - 1399.73) = -3891.33 \text{ J}$$

23. 1 mole of ideal gas initially at 10 atm and 300 K was expanded adiabatically against a constant pressure of 4 atm so as to reach equilibrium. $C_p = 28.48 + 1.76 \times 10^{-2} T \text{ J mol}^{-1}$. Calculate ΔU , ΔH and ΔS .

$$C_v = 20.17 + 1.76 \times 10^{-2} T$$

$$dU = C_v dT = PdV$$

$$\Delta U = 20.17 (T_2 - T_1) + 1.76/2 \times 10^{-2} (T_2^2 - T_1^2) = P_2 (V_2 - V_1) = P_2 (RT_1/P_1 - RT_2/P_2) = \\ R (P_2 T_1/P_1 - T_2)$$

We can now solve for T_2 .

$$T_2 = 255.3 \text{ K}$$

$$dS = C_p dT/T - R dP/P = (28.58 + 1.76 \times 10^{-2} T) dT/T - R dP/P$$

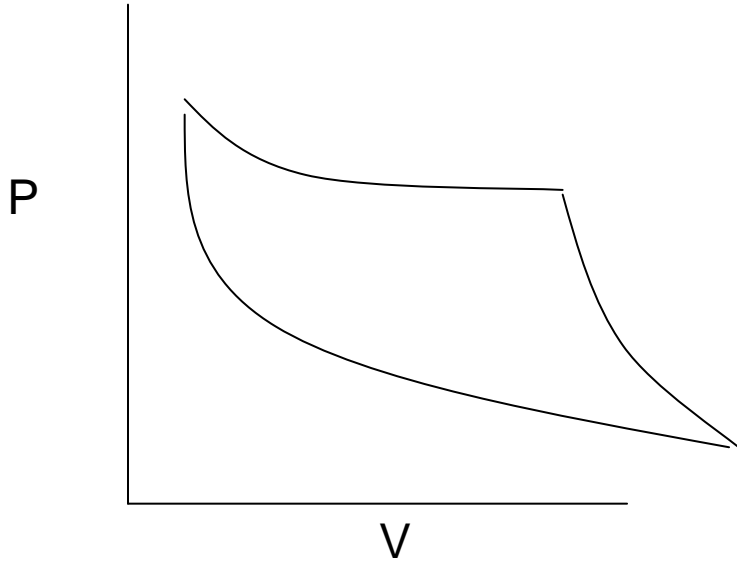
$$\Delta S = 28.58 \ln T_2/T_1 + 1.76 \times 10^{-2} (T_2 - T_1) + R \ln P_1/P_2 = 2.22 \text{ JK}^{-1}$$

$$\Delta U = \int C_v dT = -1124.3 \text{ J}$$

$$\Delta H = \int C_p dT = -1496 \text{ J}$$

24. One mole of ideal gas was subjected to the following change. $A \rightarrow B \rightarrow C$ (Details of P, T and V can be given here). Depict the above process in a P-V diagram, determine the net work done in the cyclic process. $C_v = 1.5R$.

References: Suggested books for CY101



All the details of the points A, B, C may not be given. You will have to calculate all using ideal gas law.

25. Manipulation of Maxwell's relations

26. Entropy of mixing

27. Calculation of partition functions:

Assume that 1000 particles are distributed in 5 energy levels, which are equally spaced with the spacing XX . The ground state energy is zero. What is the total energy at $T = 300$ K.

$$N = 1000$$

$$n_i/N = e^{-E_i/kT} / \sum e^{-E_i/kT}$$

Calculate n_i 's

$$E = \sum n_i E_i$$

Use of this in calculating quantities such as U .
