PATTERNS IN NATURE

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1. INTRODUCTION

A hike in the woods or a walk along the beach reveals an endless variety of forms. Nature abounds in spectral colors and intricate shapes - the rainbow mosaic of a butterfly's wing; the delicate curlicue of a grape tendril; the undulating ripples of a desert dune. But these miraculous creations not only delight the imagination, they also challenge our understanding. How do these patterns develop? What sorts of rules and guidelines, shape the patterns in the world around us?

Some patterns are molded with a strict regularity. At least superficially, the origin of regular patterns often seems easy to explain. Thousands of times over, the cells of a honeycomb repeat their hexagonal symmetry. The honeybee is a skilled and tireless artisan with an innate ability to measure the width and to gauge the thickness of the honeycomb it builds. Although the workings of an insect's mind may baffle biologists, the regularity of the honeycomb attests to the honey bee's remarkable architectural abilities.

The nautilus is another meticulous craftsman, who designs its shell in a shape called a logarithmic or equiangular spiral (explained ahead). This precise curve develops naturally as the shell increases in size but does not change its shape, ever growing but never changing its proportions. The process of self-similar growth yields a logarithmic spiral. We find the same spiral in the horns of mountain sheep and in the path traced by a moth drawn towards a light. For the mathematically inclined, such a curve can be succinctly described by the formula "R = C*(Ang), where R is the radius of the curve, C is a constant and (Ang) is the angle through which the curve has revolved. [7]

Crystals are likewise constructed with mathematical regularity. A chemist could readily explain how positively and negatively charged sodium and chloride ions arrange themselves neatly in a crystal lattice, resulting in salt crystals with a perfect cubic structure. And water molecules, high in the clouds with temperatures far below freezing, neatly coalesce to form crystalline snowflakes in the form of six-sided stars or hexagonal needles.

Next consider seashells, so often decorated with bold patterns of stripes and dots. Biologists seldom gave much thought to how these mollusks create the beautiful designs that decorate their calcified homes. Perhaps they simply assumed that the patterns were precisely specified in the genetic blueprint contained in the mollusk's DNA.

Think of the striking regularity of alternating dark and light stripes on a zebra's coat, or the reticulations on the surface of fruiting body of a morel (a vareity of mushroom) mushroom. Zooming in for a close-up of a slime mold, you can observe the branching network patterns that emerge as the mold grows. On a still smaller scale, magnified several hundred times, similar patterns emerge on the surface of a pollen grain.

The living world is filled with striped and motled patterns of contrasting colours (with sculptural equivalents of those realized as surface crests and troughs); with patterns of organization and behavior even among individual organisms. People have long been tempted to find some obscure 'intellegence' behind all these biological patterns. Even way back in early twentieth century the Belgian symbolist Maurice Maeterlinck, pondering the efficient organization of the bee and termite colonies asked:

> **" What is that governs here? What is that issues orders, foresees the future, elaborates plans and preserves equilibrium, admisters, and condemns to death? "**

Many such questions rose, increasing the curiosity to find the reason for the existence to these patterns in nature and many theories have been proposed as an attempt to do so.

2. TYPES OF PATTERNS

Though every living and non-livnig thing of the world may seem to follow a pattern of its own, looking deeply into the geometry and mechanism of the pattern formation can lead you to broadly classify them into merely two categories:

- Self-organized patterns/ Inherent organization
- Invoked organization

2.1 Self-Organized patterns [6]

One of the first cellular automata (a mechanism to study the pattern formation) to be studied in any depth was the so-called "Game of Life", devised by the mathematician Joan Horton Conway. To understand how the game of life works, imagine a huge grid of squares, entirely covered by checkers or cells, that are either black or white corresponding to 'alive' or 'dead' respectively. Each cell is surrounded by eight neighboring cells whose squares share an edge or a corner with the square occupied by the original cell. Assume that with every tick of the clock tick, the state of each cell on the entire grid evolves to its next state in accordance with four simple rules:

- A live cell surrounded by two or three live cells at time t will also be alive in the next clock tick, time t + 1 (it survives)
- A live cell with no live neighborhood or only one live neighbor at time t will be dead at time t + 1 (it dies of loneliness)
- A live cell with four or more live neighbors at time t will be dead at time t + 1 (it dies of overcrowding).
- A dead cell surrounded by three live cells at time t will be alive at time t + 1 (it will be born); otherwise a dead cell remains dead.

This theory is completely based on simple rules like above, however a slight change in any of the rules leads to predicting a completely different pattern.

To better understand how this program works, consider an even simpler version of a cellular automation (Figure 1). This one begins with only a single row (a 'one dimensional' automaton). In other words, start with a horizontal row of square cells that exceeds indefinitely far to the left and right. As in the game of life, each cell is colored either black or white. The neighborhood of each cell in the row includes just the two adjoining cells, one to its left and one to its right. And again, as in the game of life, with each tick of the clock, the color, or state, of each of the cell in the row changes according to some simple rule.

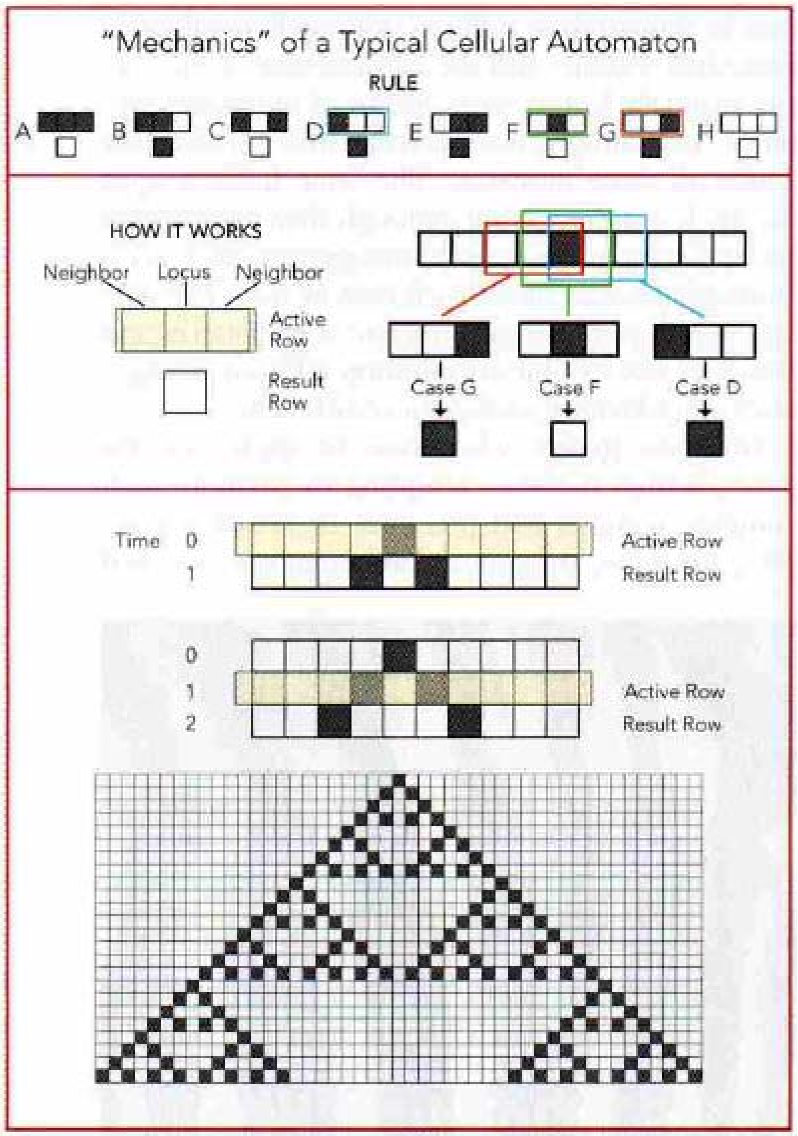


Figure 1. [6]
Example of a "One Dimensional Automaton" clearly indicating the working out on the applied rules

For example, one rule might be the following: a cell becomes black on the next tick of the clock whenever one of the other, but not both, of its neighbors are black; otherwise it remains (or becomes) white. A one-dimensional cellular automaton had the advantage that successive patterns can be represented as successive horizontal rows, the 'successor' pattern is just under its predecessor. The pattern that results is a two dimensional grid of cells that portrays the evolution of the top row throughout all the ticks of the clock. Suppose the initial row of cells has a single black cell in the centre. When the 'rule 1' (see figure) just defined is applied to that row (the active row) and then to subsequent rows, a complex pattern develops. Applying another rule to the same initial pattern would give rise to an entirely different set of successive rows, making this theory of 'game of life' highly sensitive to the laid out rules.

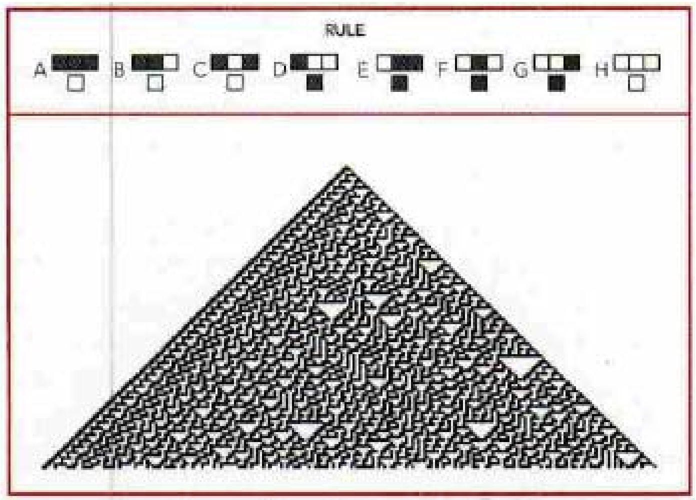


Figure 2 [6]
A completely different pattern obtained by slightly changing the rules of cell automaton

As with all self-organizing patterns, the main feature of cellular automata is that they are based on simple set of rules, and they use only local information to determine how a particular subunit evolves. But programs such as the 'game of life' or the 'one dimensional cell automaton' just described, while suggestive, lack direct biological relevance. Therefore, if rules are to be useful for understanding the patterns in life, such as the stripes on a zebra's coat, they must be different rules.

The zebra's coat alternates in contrasting areas of light and dark pigmentation. In technical jargon, the pigmentation reflects patterns of activation and inhibition - apt terms because of the dynamic process that generates the pattern. Cells in the skin called melanocytes produce melanin pigments, which are passed into the growing hairs of the zebra. Whether or not a melanocyte produces its pigment appears to be determined by the presence or absence of certain chemical activators in the skin during early embryonic development. Hence the patterns of the zebra's coat reflect the early interaction of those chemicals as they diffused through the embryonic skin.

With a new set of rules, a two-dimensional cellular automaton can readily stimulate the pattern of the coat and so shed light on the mechanism of pattern formation in the zebra. Return to the square grid and randomly place a black cell or a white cell on each square. The grid will look something like the leftmost frame in figure 3. Assume that each black cell represents a certain minimum level of pigment activator. Such a random array of activator or its absence is thought to be the starting point of early development of coat patterns.

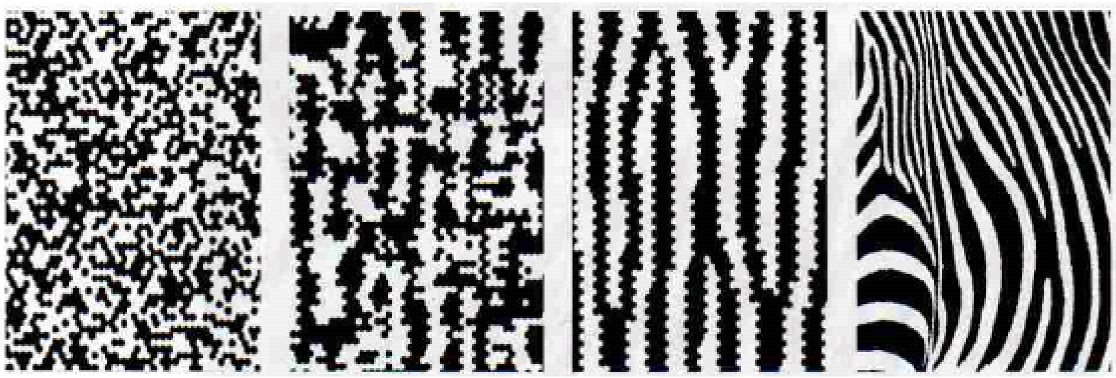


Figure 3. 2-D cell automaton for developing the zebra coat pattern [6]

Now apply another simple rule, based on the following underlying physical effect: activator molecules that are near each other strengthen and mutually reinforce their effect. At the same time they dimish the effect of activators that are far away, inhibiting their ability to activate their own nearby neighbors.

In this simple example, as in the game of life, each cell can be either on or off, i.e. black or white. And again, with each tick of the clock the cells interact with each another according to a rule that reflects the underlying physical reinforcements and inhibitions, and they switch their states appropriately. As the regions of the activator compete with one another through their local interactions, a regular pattern develops. What emerges is a self-organizing pattern that looks very much like the skin of the zebra.

Similar patterns appear in the brain. As the embryonic brain develops, competing influences from the right and left eye determine the connections that are made at the back of the brain, the visual cortex. Clusters of neutrons from one eye or the other dominate portions of the cortex in a distinct pattern. The patterns are thought to develop because the neutrons from each eye compete with one another for space. Initially, the neutronal projections coming from the left or right eye are slightly different, a difference that presumably arises at random. The rules of the competition have the same general form as the rules of activation and inhibition of zebra coat pigment. Projections of the neutrons from one eye stimulate and encourage additional projections to the area infront of the eye. At the same time those projections inhibit the projections to that area from the other eye. This local competition for real estate in the brain results in a pattern of stripes reminiscent of those of zebra.

Self-organizing patterns extends to the non-living world as well. They appear in the mineral deposits between layers of sedimentary rocks, in the path of a lightening bolt as it crashes to the ground, in the undulating ripples of windblown sand on a desert dune. When the forces of wind, gravity, and friction act on the sand dunes, the innumerable grains of sand ricochet and tumble. As one grain lands, it affects the position of the other grains, blocking the wind or occupying a site where another grain might have landed. Depending on the speed of the wind and the sizes and shapes of the grains of the sand, this dynamic process creates a regular pattern of stripes or ripples (see figure 4).

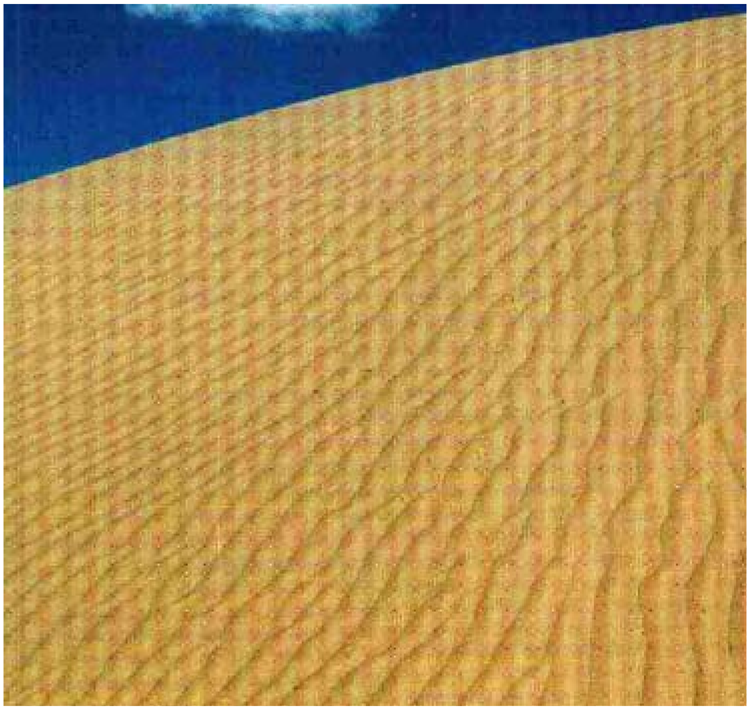


Figure 4. [6]
Stripped or rippled pattern observed on the desert sand.

Similar patterns arise accidentally on painted surfaces exposed to harsh weather. Paints and varnishes are designed to adhere permanently and evenly to the surface. Nevertheless, heat, moisture, and sunlight often combine to lift the paint off the underlying surface, causing the paint to crack or buckle. As a patch of paint begins to pull away from the surface, a dynamic tension between the forces causes the paint to buckle and wrinkle and the adhesive forces between the surfaces develops at that spot. The more paint that pulls away, the weaker the adhesive force exerted by the paint nearby that is still sticking to the surface. The result is a runaway situation but with a countervailing effect. At some point, the dynamic tensions begin to split the paint that has already pulled away. Once that happens, the tensions on the paint far from the split, still adhering to the surface, are reduced. The result is a pattern of buckling ridges.

The runaway process and its countervailing effect, so predominant in the example of the paint, are also key parts of the way the patterns form in zebra fur and sand. The runaway process is also called positive feedback; just as in a snowball rolling down a hill, more leads to even more. In the zebra, the activation near the active melanocytes increases, and so to the production of even more melanin pigment. Sand dunes develop ridges when the wind deposits a chance accumulation of sand grains. One small, almost insignificant ridge becomes amplified because it acts as a barrier, promoting the accumulation of even more grains of sand on the windward side of the ridge.

But if positive feedback operates alone and unchecked, there would be no pattern. The zebra would be entirely black; and the sand dune would have no ridges. What comes into play is a second kind of process called negative feedback, in which more leads to less. Negative feedback puts the brakes on processes with positive feedback, shaping them so as to create a pattern. The presence of an activator in the zebra skin inhibits pigment production in the nonadjacent skin patches and the zebra ends up as a mixture of black and white. A similar mechanism may also explain the uniform coat of spots in a leopard, formed from islands of high activation.

Self-organized patterns often arise in living systems because evolutionary process can build the pattern so economically. The position and branching of each and every marking of a zebra need not be explicitly specified by the limited genetic information carried by the DNA. Instead, all that's needs to be genetically coded are the characteristics of the interacting molecules. Those characteristics determine just how the molecules act upon one another- what we interpreted as the 'rules' that govern the positive and negative feedback process of the underlying activators that are distributed across the embryonic zebra's skin.

A second economy of the self-organization is an explanatory one: there is no need to invoke a different process to explain each of the many different striped and spotted patterns that occur on the surface of mammals, fish, and insects. All such patterns arise through similarly developed pathways. A particular pathway similarly emerges from the ways in which certain substances activate or inhibit one another's effects on the formation of pigment.

In non-biological physical systems, self-organized patterns are epiphenomena that have no adaptive significance. There is no driving force that pushes cloud formations, mud cracks, irregularities in painted surfaces, or spiral waves in certain chemical reactions into developing the striking patterns they exhibit. In biological systems, however, natural selection can act to favor certain patterns. The particular chemicals within the skins of the developing zebra diffuse and react in such a way as to consistently produce stripes. If the properties of the zebra skin, or the composition of the chemical activators, were even slightly different from what they are, a pattern would not develop.

2.2 Invoked Organization

Not all patterns that occur in nature arise through self-organization. A weaver bird uses its own body as a template as it builds the hemispherical egg chamber of the nest. A

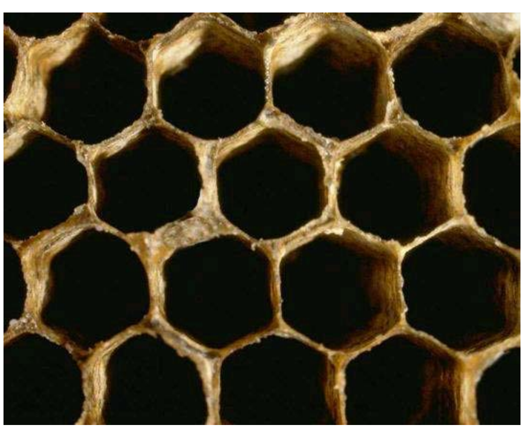


Figure 5.
Honeycomb structure is a perfect example of 'Invoked Organization'.

spider crates its sticky orb following a genetically determined recipe for laying out the various radii and spirals of the web. A caddisfly larva builds an intricate hideaway from grains of sand or other debris carefully fastened together with silk. Another very common example of invoked organization is the honeycomb made by bees (figure 5). In those cases the building of structures does involve indeed involve a little architect that oversees and imposes order and pattern. There are no 'subunits' that interact with one another to generate a pattern; instead, each of the animals acts like a stonemason, measuring, fitting, and moving pieces into place.

Finally, what about the graceful movements of the birds and fish? Do they depend on leaders, or are they also system subunits that follow 'rules' and that move above gracefully despite the absence of any leaders to guide the group. Coordinated flocking appears to rely on three behavioral rules for maintaining separation, alignment, and cohesion with the nearby birds; maintain the average heading of nearby birds; and move toward the average position of nearby birds. Fishes rules are similar, and they suffice to describe the phenomenon.

It is not easy for human beings to intuit how such a decentralized mode of operation can function so effectively, because human groups rely so heavily on hierarchical organization. Executive functioning, planning, and decision making exist at many levels of the hierarchy. Imagine a world without supervisors, administrators, and managers, and many people would imagine sheer chaos. Nevertheless, self-organization in nature is efficient, economical, and ubiquitous. It is one of the least known, yet most powerful, devices for achieving pattern and order in the world.

3. ELEGANT PATTERNS FROM COMPUTER MODELS

Next consider seashells, so often decorated with bold patterns of stripes and dots. Biologists seldom gave much thought to how these mollusks create the beautiful designs that decorate their calcified homes. Perhaps they simply assumed that the patterns were precisely specified in the genetic blueprint contained in the mollusk's DNA. But some years ago, scientists, skilled in both biology and computer science, began to look at pattern formation in an exciting new way. One of the first things they realized was that two individuals of the same species were similar, but not identical. Like the fingerprints on one's hand, they are alike yet not alike. This simple observation led them to hypothesize that the patterns on shells, the stripes on a zebra, and the ridges on our fingertips are not rigidly predetermined by the genetic information inside the cell's nucleus. Organisms are not built as a house is built, by meticulously following an architect's plans. Instead, genes appear to take a more generalized approach, specifying sets of basic rules whose implementation results in organized form and pattern. Tackling the problem of how markings develop on shells, these scientists proposed a few simple rules for how pigment precursors in cells might diffuse along the snail's mantle at the growing edge of the shell. Then, by repetitively implementing these simple rules in a series of computer simulations, they "created" shell patterns with a startling similarity to real shells. These scientists readily admit that this similarity does not prove that shell patterns develop in the manner they hypothesize, but it does suggest that simple mechanisms could account for some of the complex and varied patterns observed in nature.

Over the years, these same ideas have been applied to many questions in developmental biology concerning how structures become organized. One of the greatest biological mysteries yet to be solved is how a single egg apparently devoid of structure - becomes a child. The human cell does not contain enough information to specify the location and connections of every neuron in the brain. Therefore, much of the body's organization must arise by means of more simple developmental rules. In nature many systems display extreme complexity, yet their fundamental components may be rather simple. The brain is an organ of unfathomable complexity, but an isolated neuron cannot think. Complexity results from interactions between large numbers of simpler components. With the advent of powerful computers, mathematicians, chemists, physicists, biologists have begun to discover how simple interactions between large numbers of subunits could yield intricate and beautiful patterns. Suddenly people are studying all sorts of phenomena both mundane and bizarre - piles of sand, dripping water faucets, slime molds, leopard's spots, forest fires, flocking birds and visual hallucinations. Though these various phenomena have little in common, they are all fertile subject matter for those who study nature's complexity. And this emerging field has given us a new vocabulary including such terms as chaos, fractals and strange attractors.

4. MATHEMATICS: AS A SOLUTION TO THE PATTERN FORMATION

The geometry of most patterns in nature can be linked to mathematical numbers either directly or indirectly. Though, for some cases, these relations seem to have been forced through, the high degree to which natural patterns follow mathematical series and numbers is amazing. However, to understand this correspondence, it is first necessary to have an appreciable knowledge of a few mathematical series, ratios and plots.

4.1 The Golden Ratio and the Fibonaccci Series [11]

Leonardo Fibonacci began the study of this sequence by posing the following problem in his book, Liber Abaci

> **"How many pairs of rabbits will be produced in a year, beginning with a single pair , if in every month each pair bears a new pair which becomes productive from the second month on?"**

Of course, this problem gives rise to the sequence 1, 1, 2, 3, 5, 8, 13, ... in which any term after the first two can be found by summing the two previous terms. In functional notation we could write $f(n) = f(n - 1) + f(n - 2)$ using $f(0) = 1$ and $f(1) = 1$. The ration between two consecutive terms of this series tends to the number 1.61803399. This numbers is not as simple as it looks,. It is a number commonly encountered when taking ratios of distances in simple geometric figures such as pentagons, decagons and dodecagons. It is denoted by **PHI**, and is called the divine proportion, golden mean, or golden section.

Most people are familiar with the number Pi, since it is one of the most ubiquitous irrational numbers known to man. But, phi is another irrational number that has the same propensity for popping up and is not as well known as Pi. This wonderful number has a tendency to turn up in a great number of places, a few of which will be discussed below.

One way to find Phi is to consider the solutions to the equation

$$X^2 - X - 1 = 0$$

When solving this equation we find that the roots are

$$X = (1 \text{ (+ or -) } 5^{1/2})/\ 2$$

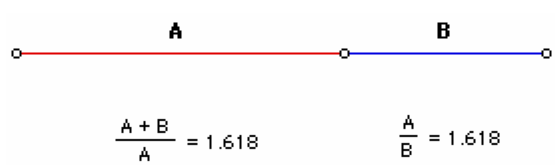
We consider the first root to be Phi. We can also express Phi by the following two series

$$\phi = [1, 1, 1, \ldots] = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

and

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$$

Phi can also be found in many geometrical shapes, but instead of representing it as an irrational number; we can express it in the following way. Given a line segment, we can divide it into two segments A and B, in such a way that the ratio of the length of the entire segment is to the length of the segment A is same as that of the length of segment A is to the length of segment B. If we calculate these ratios, we see that we get an approximation of the Golden Ratio.



Another geometrical figure that is commonly associated with Phi is the Golden Rectangle. This particular rectangle has sides A and B that are in proportion to the Golden Ratio. It has been said that the Golden Rectangle is the most pleasing rectangle to the eye. If fact, it is said that any geometrical shape that has the Golden Ratio in it is the most pleasing to look at of those types of figures.

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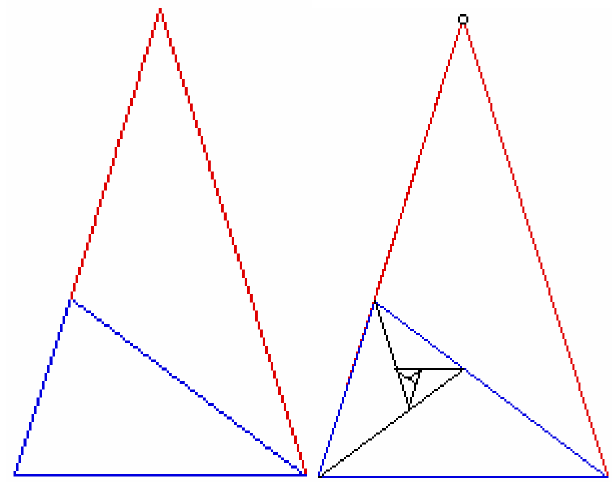


Figure 6. Sequence of steps to form the logarithmic curve from isosceles triangles – based on the golden section.

Let's turn back to one of the Golden Triangles for a moment. If we take the isoceles triangle that has the two base angles of 72 degrees and we bisect one of the base angles, we should see that we get another Golden triangle that is similar to the first. If we continue in this fashion we should get a set of Whirling Triangles.

Out of these Whirling Triangles, we are able to draw a logarithmic spiral that will converge at the intersection of the two blue lines in Figure 3.

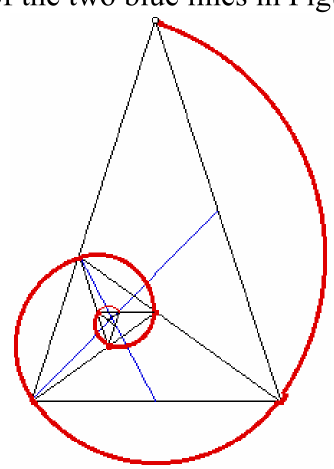


Figure 7. A logarithmic spiral – a commonly observed pattern in nature

We can do a similar thing with the Golden Rectangle. We can make a set of Whirling Rectangles that produces a similar logarithmic spiral. Again this spiral converges at the intersection of the two blue lines, and this ratio of the lengths of these two lines is in the Golden Ratio.

4.2 Phyllotaxis [12]

Patterns in nature that relate to numbers are collectively known as phyllotaxis. The most interesting examples of phyllotaxis are related to the Fibonacci series and the golden section. Few examples have been mentioned below.

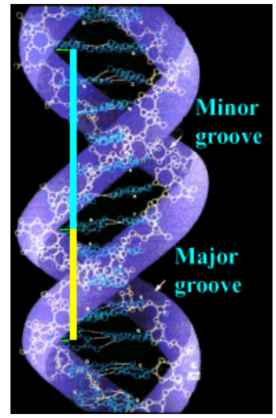
- The human face abounds with examples of the golden section or the divine ratio. The head forms a golden rectangle with the eyes at its midpoint. The mouth and the nose are each placed at golden sections of the distance between the eyes and the bottom of the chin. You can draw a perfect square having two of its four corners at the two pupils of the eyes and the remaining two at the corners of the mouth. The golden section of the four sides to the square gives the nose, the inside of the nostrils, the two rises of the upper lip and the inner points of the ear. Note, that the length of the square is same as the distance of the upper lip to the bottom of the chin. The two front incisor teeth form a golden rectangle, with a phi ratio in height to width. The ration of the width of the first tooth to the width of the second tooth is also phi. The distance of the third tooth from the centre of the mouth is the golden section of half the width of the smile.

- The rest of the human body illustrates the divine proportion or the golden section as well. The height of the head with reference from the finger tips of arms stretched down is a golden section of the total height of the body. The distance of the head to the shoulders is a golden section of this height. The distance of the head to the navel is also a golden section of the total height.
  Consider the index finger of the hand. It is divided into three visible parts (2 joints). Looking straight at your finger (pointing up), you can easily prove that the ratio of the bottom part to the middle part is equal to the ratio of the middle part to the top part, and both are equal to phi.

- Phi has also been used by mankind for centuries in architecture. It all started as early as with the Egyptians in the design of the pyramids. The 'Greeks' have also commonly used the divine ratio for the beauty and balance in the design of the Parthenon. Renaissance artists from the time of Leonardo Da Vinci knew it as the divine proportion and used it to design the Notre Dame in Paris. The CN Tower in Toronto, the tallest tower and freestanding structure in the world, contains the golden ratio in its design (the ratio of the observation deck height to the total deck height is equal to phi). It is mathematically been proven to be the most appropriate ratio for stability.

- The DNA molecule, the program of life, is also based on the golden section. It measures 34 angstroms in length and 21 angstroms wide for each full cycle of its double helix spiral. 34 and 21, of course, are numbers of the Fibonacci series and their ratio is almost equal to Phi.

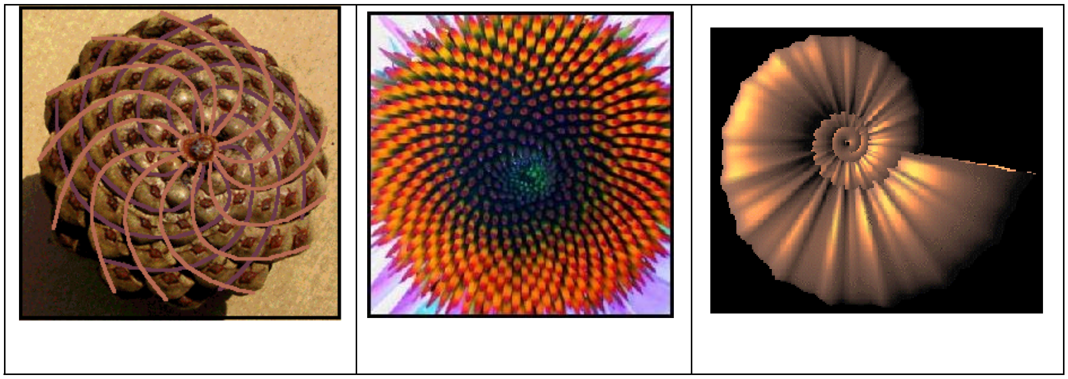
Figure 8. A DNA molecule based on the principle of phi



- The spiral growth of the sea shell follows the logarithmic spiral, which is completely based on the golden rule. See figure below.

- Most crystals in nature, such as those in sugar, salt or diamonds, are symmetrical and all have the same orientation throughout the entire crystal. Quasicrystals represent a new state of matter that was not expected to be found, with some properties of crystals and others of non-crystalline matter, such as glass. With five-fold symmetry, once thought of be impossible, they were first observed in 1984 in an aluminium-manganese alloy (Al6Mn). Since then, quasicrystals have been found in other substances. "Penrose tiles" allow a two-dimensional area to be filled in five-fold symmetry, using two shapes based on Phi. It was thought that filling a three-dimensional space in five-fold symmetry was impossible, but the answer was again found in Phi.

- Plants illustrate the Fibonacci series in the numbers and arrangements of petals, leaves, sections and seeds. Plants that are formed in spirals, such as pinecones, pineapples and sunflowers, illustrate Fibonacci numbers. Many plants produce new branches in quantities that are based on Fibonacci numbers.



(a) (b) (c)

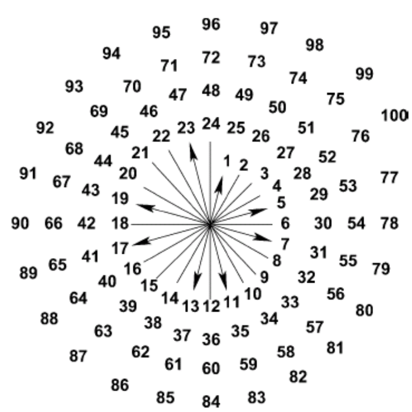
Figure 9. (a) A pine cone exhibits the pattern of spirals of both directions – 13 clockwise and 8 anticlockwise (13 and 8 are consecutive terms of the Fibonacci Series)
(b) The seed of the cone flower following a logarithmic spiral pattern
(c) The shells of snails are also in the shape of spirals.

Jan Boeyens [4] has related structure and periodicity of atomic numbers with natural number sequences. He stated that by arranging the natural numbers along a spiral with a period of 24, you will get all the prime numbers along radial lines, i.e. they lie on eight arms on the so called prime-number cross, marked by arrows in the figure. This eight-line cross pertains to the periodicity of the chemical elements by the following property.

$$\sigma_n = \sum_{i=24n}^{24(n+1)} i = (2n+1)a; \quad n = 0, 1, 2 \ldots; \quad a = 300$$

The coefficient (2n+1) matches the degeneracy of the angular momentum wave functions of the hydrogen electron. Therefore the above sum "300 (2n+1)" corresponds to the total number of electron pairs over all elements for a fixed number of atoms. Logically speaking, instead of 300, we should have the total number of unique elements (approx. 1/3rd of 300), but 300 actually includes all the different isotopes of the unique elements which are also expected to obey the periodic law.

Figure 10. Natural numbers arranged on a spiral of period 24



Boeyens has also given the natural numerology of the DNA code, obtained on the basis of four number systems, such as 4-ary gray code. Gray code is a system that represents integers as bit strings in such a way that from one integer to the next only a single bit changes in their representation. In binary, more than one bit can change, e.g. 7=111, whereas 8=1000. To go from one integer to the next in Gray code, the value of the bit in the least significant digit changes **(0 <-> 1)** to give a bit string not used already. Simple algorithms exist to generate generalized Gray code of integer N directly from the base n representation. Exactly as for Gray code, a single digit changes between successive integers represented by generalized Gray code. There is a natural relationship between 4-ary Gray code and the DNA codons3 which can be understood by making the following correspondences: 0 - C, 1 - A, 2 - G, 3 - U.

Mindful of the fact that the 64 DNA triple-base codons code for 20 amino acids and one stop, while exactly 21 numbers occur on the cross between 0 and 63, it may be instructive to arrange the codons in 4-ary Gray code in sequence along the number spiral of previous figure. The result is shown in above.

Although the correlation between amino acids and prime-cross numbers is not perfect, it is significant. The present composition of amino acids in proteins is likely a product of an evolutionary process that has refined a more primitive code. In fact, Jukes proposed an archetypal code for only 16 amino acids: in this scheme, each group of four codons in the figure codes for a single amino acid to yield a one-to-one correspondence between amino acids and circled numbers. The important result is the similarity between a prime-number distribution and both elemental periodicity and DNA code.

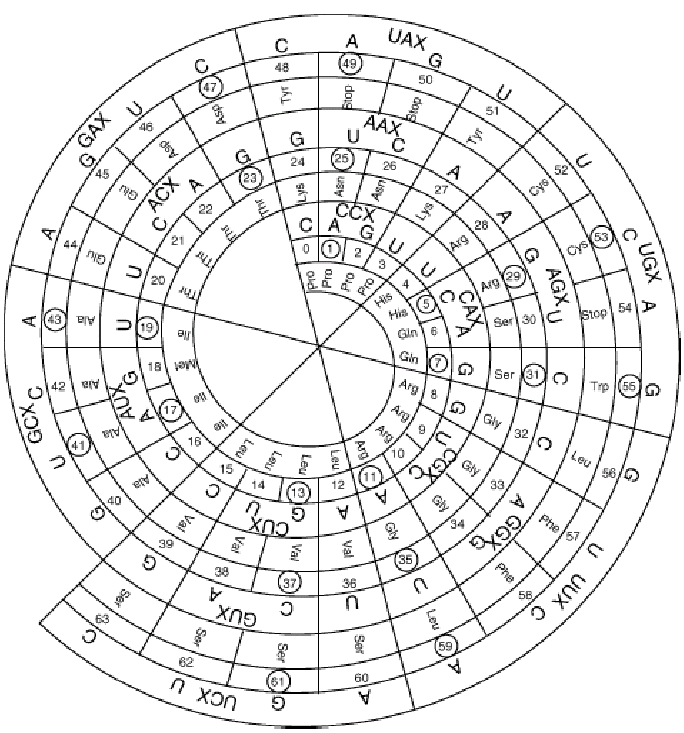


Figure 11. DNA codons, arranged in 16 groups of four along the number spiral, in sequence of their 4-ary Gray-code equivalents, from CCC=0 to UCC=63. Numbers on the prime cross are circled. Amino acids coded for by each triplet are shown alongside of the Gray code numbers.

5. BIOLOGICAL PATTERN FORMATION [2]

The generation of the complex structure of a higher organism within each life cycle is one of the most fascinating aspects of biology. The necessity of mathematical models for morphogenesis is evident. Pattern formation is based on the interaction of various components. Turing (1952) showed that under certain conditions two interacting chemicals can generate a stable inhomogeneous pattern if one of the substances diffuses much faster than the other. However, this is against “common sense”, since diffusion is expected smooth out concentration differences rather than to generate them. Since then, quite extensive work has been done in generating biochemically more feasible models, which can be applied to different situation. The relevance of chemical gradients in biological systems in pattern formation and cell differentiation is high. The patterns that can be generated are graded concentration profiles, local concentration maxima, and stripe like distributions of substances. In many developmental systems small regions play an important role because they are able to organize the fate of the surrounding tissue. The local concentration of a substance that is distributed in a graded fashion governs the direction in which a group of cells has to develop.
The theoretical models developed for pattern formation have to give satisfactory answers for the following questions:

- How does a system maintain large scale inhomogeneities (gradients) even when they originate from homogeneous conditions?
- How do cells measure the local concentration to find their position in a gradient and choose the direction to develop?

5.1 SIMPLE MODELS FOR PATTERN FORMATION

Gierer and Meinhardt (1972) and independently Segel and Jackson (1972) have shown that two features play a central role: **local self-enhancement and long-range inhibition**. Self- enhancement is essential for local inhomogeneities to be to be amplified. A substance a can be called autocatalytic or self-enhancing if a small increase in concentration of a in steady state homogeneous condition results in further increase of a. it can also result from a substance b which assists formation rate of a. self enhancement in itself is not sufficient to generate a pattern as once formation of a starts it will result in formation of a large scale homogeneous structure. Therefore it has to complement by an action of a fast diffusing antagonist or inhibitor. This results in formation of large scale inhomogeneous structures.

Two types of antagonist reactions are plausible:

- Either an inhibitory substance h is produced by the activator that, in turn, slows down the activator production
- A substrate s is consumed during autocatalysis. Its depletion slows down the self-enhancing reaction.

Gradient-Formation

A pattern emerges whenever the size of the field becomes larger than the range of the activator. In fields with a size comparable to the activator range, the high activator concentration can be formed at one end of the field only

5.1.1 ACTIVATOR- INHIBITOR SYSTEM

The following equations describe the interaction between activator **a** and inhibitor **h**:

$$\frac{\partial a}{\partial t} = \frac{\rho\, a^2}{h} - \mu_a a + D_a \frac{\partial^2 a}{\partial x^2} + \rho_o$$

$$\frac{\partial h}{\partial t} = \rho\, a^2 - \mu_h h + D_h \frac{\partial^2 h}{\partial x^2}$$

These depend on the production, removal and exchange rates with neighboring cells (diffusion) and a small baseline (activator independent) production rate. The latter is required to initiate the reaction, for instance during regeneration or during oscillations

$$\rho = \rho_a \frac{1}{(1 + \kappa_a a^2)}$$

ρ_a = the cross-reaction coefficients

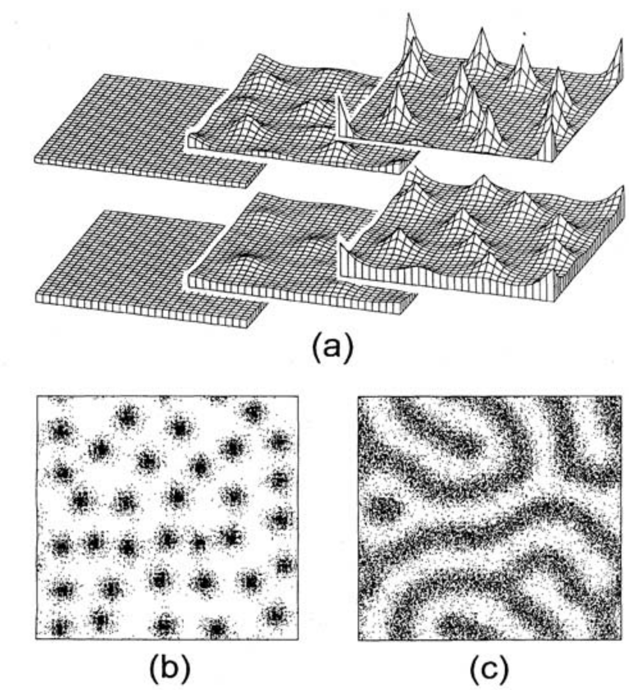


Figure 12. Pattern produced by activator inhibitor model (a) initial, intermediate and final stage: activator (top) – inhibitor (bottom). (b) Simulation in large space. (c) Saturation can lead to stripe like formation.

The saturation constant K_α, has a deep impact on the final aspect of the pattern. Without saturation, somewhat irregularly arranged peaks are formed whereby a maximum and minimum distance between the maxima is maintained [Figs. 12(a) and l2(b)]. In contrast, if the autocatalysis saturates (K_α > O), the inhibitor production is also limited. A stripe like pattern emerges

5.1.2 ACTIVATOR-SUBSTRATE SYSTEM

The antagonistic effect may result from a depletion of a substrate or co-factor that is required for the self-enhancing reaction. The following equations describe the interaction

$$\frac{\partial a}{\partial t} = D_a \triangle a + \rho_a \left(a^2 s - a \right)$$
$$\frac{\partial s}{\partial t} = D_s \triangle s + \rho_s \left(1 - a^2 s \right)$$

This model has some different properties as compared to that shown by the activator inhibitor model. As is shown in figure comparatively rounded peaks are observed by this model as compared to the later

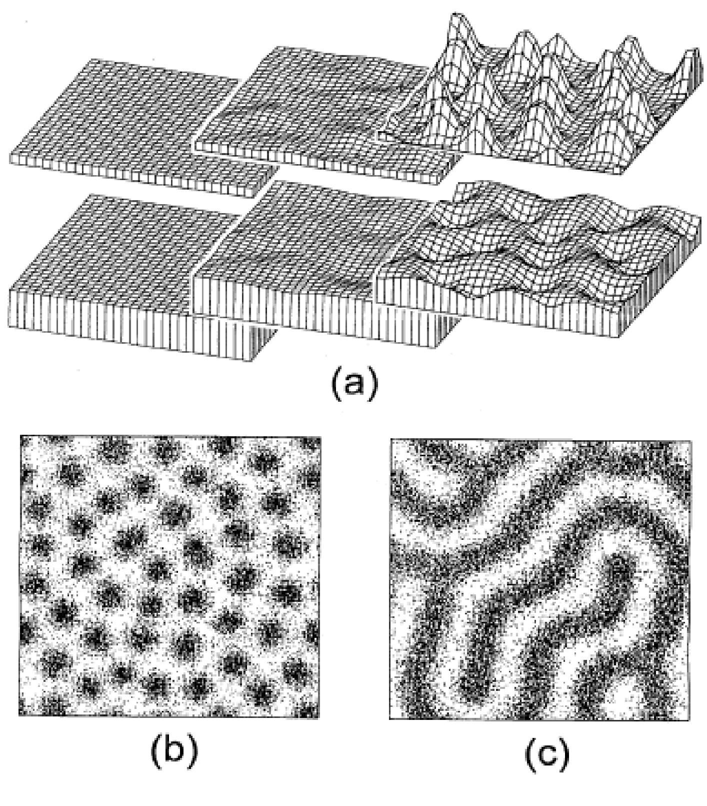


Figure. 13 Patterns produced by the activator-substrate model (a) initial, intermediate and final stage: activator (top) – substrate (bottom). (b) Simulation in large space. (c) Saturation can lead to stripe like formation.

In a growing field of cells, an (**a, s**) system produces new maxima preferentially by a split and shift of existing ones; while in (**a, h**) models new peaks are inserted at the maximum distance from the existing ones. The reason for this shift can be explained as follows. With growth, the concentration of **s** increases in areas between the activator maxims this leads to faster formation of **a** at the sides of a maximum as compared to its centre. This results in shifting of maxima until an optimum balance is reached. If a maximum has to be displaced or to form a wave, one will preferentially use an (**a, s**) system. In contrast, if an isolated maximum has to be generated, we shall employ an (**a, h**) system.

5.1.3 BIOCHEMICAL SWITCHES

In an organism growing beyond this size, cells have to make use of the signals they have obtained by activating particular genes.

5.1.4 OTHER MODELS:

$$\frac{\partial a}{\partial t} = D_a \triangle a + \rho_a \left(\frac{c}{1 + \kappa_a b^2} - a \right) + \sigma_a$$

$$\frac{\partial b}{\partial t} = D_b \triangle b + \rho_b \left(\frac{1}{1 + \kappa_b a^2 c} - b \right) + \sigma_b$$

$$\frac{\partial c}{\partial t} = D_c \triangle c + \rho_c \left(b - ac \right) .$$

In this example the two substances **a** and **b** mutually repress each other's production. A small local increase of **a** leads to a decrease in production of the **b**. If **b** shrinks, **a** increases further, and so on. In this case, self enhancement results from the local repression of **a**. The necessary long-range inhibition is mediated by, the rapidly diffusing substance **c**, which is produced by, **b** but is poisonous to it. Further, **c** is removed with the, help of **a**. So, although **a** and **b** are locally competing, **a** needs **b** in its vicinity and vice versa. Therefore the, preferred pattern generated by such a system consists of stripes of **a** and **b**, closely aligned with each other.

5.2 FORMATION OF PERIODIC STRUCTURES

5.2.1 INSERTION OF NEW MAXIMA DURING GROWTH:

A steady state pattern formed from activator inhibitor model will consist of irregularly arranged peaks. Due to the lateral inhibition each peak is separated by some minimum distance. Now as the field grows, the distance between the maxims increases and the concentration of inhibitor decreases. This results in activation which leads to formation of a new peak. Therefore density and overall spacing of maxims remains constant.

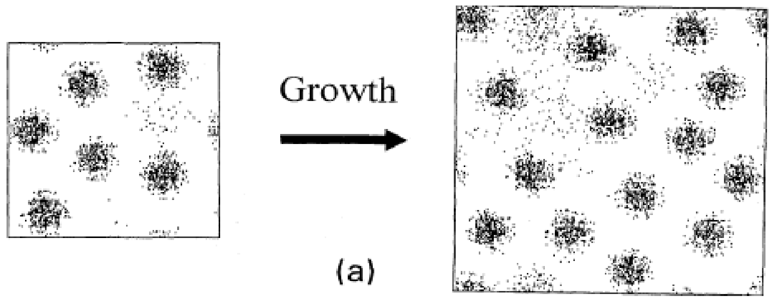


Figure 14(a) insertion of new maxima during growth. The figure is made with (a, h) model and shows activation concentration.

5.2.2 STRICTLY PERIODIC PATTERN FORMATION

The periodic pattern becomes more regular if the pattern forming reaction works already during growth. With the addition of new cells at the boundaries, the distance between these cells and the existing maxima increases and the inhibitor concentration decreases. Whenever the inhibitor concentration becomes lower than some threshold a new maximum is triggered. Therefore, whenever the distance to an existing maximum becomes large enough, a new one is inserted, leading to formation of a strictly periodic structure.

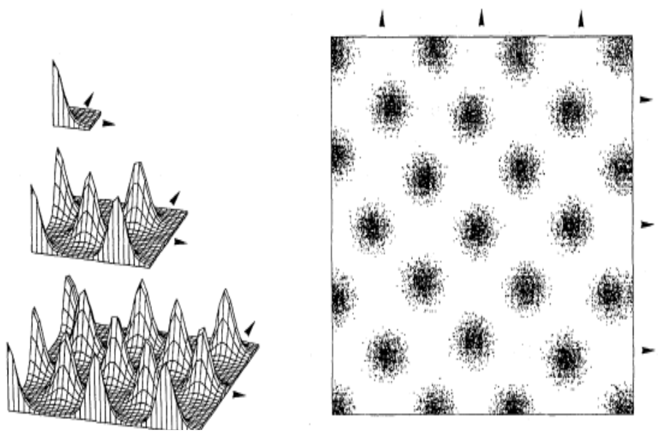


Figure. 15 Generation of periodic structures during marginal growth. The arrows represent the direction of growth. The figure is made with (**a, h**) model

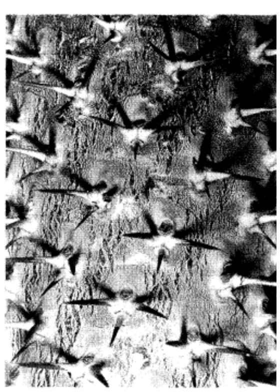


Figure. 16 Spacing of thorns on cactus. This formation can be made by using the above model.

5.3 FROM SIMPLE GRADIENTS TO COMPLEX STRUCTURES

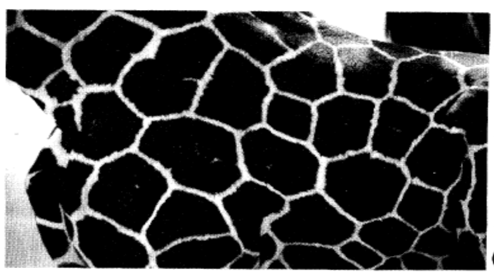
Now that we can generate inhomogeneous systems out of initial uniform systems combining several systems of such kind may lead to the formation of very complex structures. A first system A modifies the system and triggers a second system B. Reversal From B to A is assumed to be weak. However there are a few points to be taken care of :

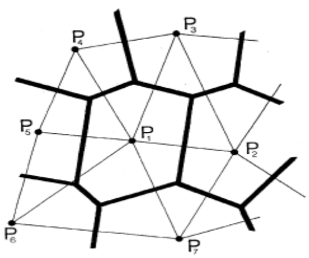
- The second system has to respond dynamically to any changes in the first one, i.e. any changes in A must be transferred to B.
- A has to just trigger B; the structure formed by B is stable even after A vanishes.

In the examples mentioned below we will use the rules mentioned above:

5.3.1 Animal coatings

A simple reaction-diffusion mechanism is used to explain some variety of patterns ranging from spots on cheetah to reticulated coat of the giraffe. In mammals, hair pigmentation is due to Melanocyte, which is supposed to be uniformly distributed in the derma. Whether they produce melanin (which colors hairs) or not is believed to depend on the presence of some unknown chemicals whose pattern is laid down during the early embryogenesis (Bard, 1977).


(a)



(b)

Figure. 17 Analogy of giraffe pattern and Dirichlet domains.(a) side of a giraffe (b) Dirichlet polygon

The Figure.17 shows similarity between a polygonal spots that cover the animal and Dirichlet domain suggesting that diffusion mechanism in animal leads to formation of Dirichlet polygons. Consider a surface S and points $P_1, \ldots, P_n$, randomly scattered on it. Suppose that each P_i, initiates at a given time a chemical wave, which spreads uniformly in all directions. The system should be such that, if two waves meet, they annihilate each other. The lines along which annihilation occurs define the envelopes of the Dirichlet domains around the initial centers P.

This uses a activator-substrate (**a, s**) model combined with a switching system y. Melanocyte activity is given by **y**, where **y = 1** corresponds to cells producing melanin, while **y = 0** corresponds to cells that do not. Initially concentration of **a** is zero everywhere except for a few places. This high value of a switches **y = 1**. On the other hand, due to the depletion of **s** and to its low diffusion constant D, high-**a** regions shift toward zones where the substrate is abundant: **a** waves propagate over the surface. When two such waves get close, they annihilate each other due to the depletion of substrate **s**.

since y needs **a to** trigger it, **y** becomes zero. Thus giving rise to formation of Dirichlet polygons.

5.3.2 The fine veins of the wing of a dragonfly

The above pattern once formed is fixed and no new lines can be inserted during growth to subdivide a large polygon into two smaller ones. This is suitable for the polygonal pattern observed in giraffe but not for the wing of a dragon fly. For such systems it is expected that the final pattern is not formed at a particular moment in the development stage. We assume that the main veins of a fly are genetically determined and the finer ones are presumably added later in order to strengthen the growing structure and to keep approximately constant the size of a domain enclosed by veins.

It consists of a (**a, s**) activator substrate system and a (**b, h**) activator inhibitor system. The concentration of a modifies the saturation value of b. Thus high values of **a** sets off the (**b, h**) system. In regions with low **a**, the process is reversed, (**b, h**) system triggers leading to formation of a stripe-like boundary. This effect is further enhanced by **s** due to increase in formation of **b**.

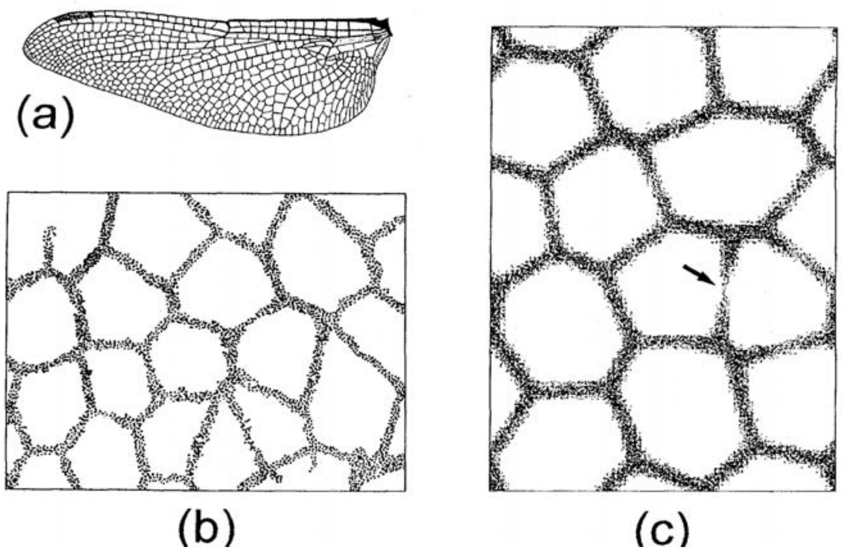


Figure.18 Polygon structures: (a) the left posterior wing of the dragon fly. (b) Schematic of fly veins. (c) Completion of a new boundary between the two domains (arrow)

Thus, stripes will appear along sites with a high concentration of **s**, in regions that are most distant from the maxima of **a**. Due to the action of **h**, the stripes become sharp. New boundaries are inserted whenever the system becomes too large. This is because, with growth, the distance between the maxima increases. If a certain distance is surpassed, **a** maximum splits into two and displacement towards a higher substrate concentration follows. Between these two maxima, a new region with high substrate concentration appears that, in turn, initiates a new **b** line (Figure. 18).

6. CONCLUSION

The world around us seems to make up of several distinct patterns, evolving various complex steps of formation. However, looking more deeply we see many similarities and resemblances. The numerous models explained above have no experimental proof and may not be correct, but they definitely show linkages between patterns formed under highly contrasting natural conditions e.g. (a zebra coat and sand dunes) and also show that the mechanisms between the formations of these patterns need not necessarily be complex.

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